Markov Processes

A Markov process is a stochastic extension of an FSM. State transitions are probabilistic, and there is no input to the system. At each time step, the system is only in one state.

Process diagrams are a natural way of representing Markov processes.

A Markov chain is a sequence of random variables X1,X2,... with the Markov property, that the probability of any given state Xn depends only on its previous state Xn-1.

P(Xn= x | Xn-1 = xn-1,....X1 = x1) = P(Xn= x | Xn-1 = xn-1)

The possible values of Xi make the countable set S called the state space of the chain. If the state space is finite, and the Markov chain is time-homogenous (transition probabilities are constant), the transition probability can be represented by the transition matrix P.

A probability distribution converges towards a stationary distribution when

x\* = lim(n->infinite) x(n)

And x\* = P x\*

A hidden Markov model is a model where the system is assumed to be a Markov process with unknown parameters. Each hidden state has an emission probability, which can be observed.

To find the most likely state sequence, the Viterbi algorithm is used. In each step, we keep track of

1. The relative probability
2. The most probable sequence of hidden states so far

The complexity of this algorithm is O(M2N) where M = |S| and N is the number of observed outputs.

A Markov chain is a regular Markov chain if some power of the transition matrix has only positive entries.

Some properties of regular Markov chains are

1. Pn->W as n->infinite where W is a constant matrix and all of W’s columns are same
2. There is a unique probability vector w such that Pw = w.
3. Pnp0->w as n->infinite for any probability vector p0.

A state Sk in a Markov chain is called an absorbing state if once the Markov chain enters the state, it remains there forever. This means the self transition probability is 1.

A Markov chain is an absorbing chain if

1. It has at least one absorbing state
2. For every state in the chain, the probability of reaching an absorbing state in a finite number of steps is nonzero

The non-absorbing states are called transient states.